

Intergroup Competition for the Provision of Binary Public Goods

Amnon Rapoport

University of North Carolina—Chapel Hill

Gary Bornstein

Hebrew University of Jerusalem, Israel

An experimental paradigm is proposed for investigating interpersonal conflicts under conditions of intergroup competition. The paradigm constitutes an n -person game with imperfect and incomplete information in which the n players are divided into two groups, are each assigned monetary endowment, and must then decide independently and anonymously whether to keep the endowment or contribute it to the group's benefit. The group with the larger sum of contributions is provided with a monetary reward (binary public good), which is shared equally among all of its members regardless of their decision. Two alternative models are proposed and their testable implications are derived and discussed. Both assume maximization of expected utility, but their assumptions about the expectations each player holds about the decisions of the remaining players are different. The effects of predecisional communication are examined and several extensions of the basic paradigm are outlined.

Although the interest of social psychologists in intergroup relations has been growing rapidly (Billig, 1976; Brewer, 1979; Tajfel, 1982), the experimental research in this area is still sparse, especially when compared with the vast amount of research on interpersonal relations (Rabbie, 1982). The difficulty of simulating complex intergroup relations in the laboratory is one reason for the field's slow development (Gerard & Miller, 1967; Rabbie, 1982). Another and perhaps more important reason appears to be the lack of an integrated theory. Thus, Rabbie (1982) wrote:

Although the literature offers a number of hypotheses about intergroup relations, these hypotheses do not form an integrated theory. They are usually stated at one particular level of analysis, they are sometimes contradictory, and their supporting evidence varies widely. (p. 124)

The development of an integrated theory has been especially hindered by the traditional distinction between the interpersonal and intergroup levels of analysis. This distinction has characterized the theoretical study and experimental investigation of social conflicts and competitions for many years (Tajfel, 1982). Despite recent attempts to simultaneously investigate the interpersonal and intergroup aspects of social conflicts in small groups (Brewer & Kramer, 1986; Kramer & Brewer, 1984), the literature shows little success in closing the gap between these two largely hypothetical levels of analysis.

One possible way to narrow the gap is by designing and conducting systematic and theory-driven research of social situations involving small groups, in which social conflicts occur simultaneously at both levels of analysis. Extending the paradigm

studied by van de Kragt, Orbell, and Dawes (1983) and building on the expected utility models proposed by Rapoport (1985, 1987) for this paradigm and its various modifications, the major purpose of our study is to propose an experimental paradigm and a psychological theory for interpersonal conflicts in small groups under conditions of intergroup competition. Specifically, we will address the problem of interpersonal interaction in situations of intergroup conflicts concerning the provision of binary (step-level) public goods. The next four sections (a) discuss the problem of public good provision in intergroup conflicts; (b) relate it to the problem of binary good provision in interpersonal settings; (c) present an experimental paradigm for studying the variables that affect the decision to contribute or not; and (d) propose two variants of an expected utility model that relate such decisions to the payoff structure, group size, and expectations each member of the two competing groups has about the decisions of the other members. In subsequent sections we investigate the effects of predecisional communication on the individual decisions to contribute or not and discuss various testable implications of the models.

Intergroup Competition as Public Goods Problems

Intergroup conflicts are often centered on the acquisition of scarce resources (e.g., grazing land, monetary prizes) for which the groups compete. However, group competition may also occur when the scarce resources have no value outside the context of the competition itself. This is the case when groups compete to achieve higher rank, enhance their prestige, or win a contest, examples of "social competition" as Turner (1975) has named it. Moreover, the study of minimal groups suggests that even when there is no explicit or institutionalized competition, groups tend to compete over positive social identity (Tajfel, 1982).

Regardless of whether the scarce resources for which the groups compete are material or social and whether the ensuing conflict is designated as objective or social competition (Tajfel, 1982), the benefits associated with winning the competition are

Preparation of this article was supported by grants from the Israel Foundation Trustees and the Faculty of Social Sciences at the Hebrew University of Jerusalem.

Correspondence concerning this article should be addressed to Amnon Rapoport, Department of Psychology, University of North Carolina, Davie Hall 013A, Chapel Hill, North Carolina 27514.

often shared jointly by the members of the winning group. Exclusion of individual members from sharing these benefits, regardless of the level of their contribution to their group's success, is often impossible. This is the case, for example, with the tug-of-war contest, in which two teams pull at either end of the rope until the losing team is drawn over the center line. The individual contribution to the group's success or failure in this contest may not be easily assessed. Another, more familiar example is crew racing. Work crews who compete for status, vacation, or some other predetermined prize are yet a third example.

On the basis of the two defining properties of jointness of supply and impossibility of exclusion, the group's benefits in such competitions constitute public goods (e.g., Barry & Hardin, 1982). The fundamental problem each group member faces when his or her decision is private and anonymous is whether to contribute (if contribution is binary) or how much to contribute (if contribution is continuous). Two predominant motives for either not contributing (when contributing is binary) or lowering the level of contribution (when contribution is continuous) have been identified. The first is the fear of having wasted one's contribution when one's group eventually loses the competition, and the second is the opportunity of a free ride.

A public-goods problem in which a sufficient level of contribution results in the provision of the public good and an insufficient level results in its not being provided is characterized as a problem of step-good collective action (R. Hardin, 1982). Although continuous provision of public goods may be appropriately modeled as an n -person Prisoner's Dilemma game in which the strategy not to contribute is unconditionally best, a dominant strategy can no longer be identified for step-good problems (Frohlich & Oppenheimer, 1978; R. Hardin, 1982). Rather, the decision whether to contribute depends on the assumptions one has about the behavior of others. It was this particular aspect of step-level public goods—the lack of a dominating strategy—that led van de Kragt et al. (1983) to develop their experimental paradigm and Rapoport (1985, 1987) to propose his models.

Interpersonal Competition for the Provision of Binary Public Goods

Apparently motivated by the work of Campbell (1975) and G. Hardin (1968, 1977), who have viewed with skepticism or mistrust solutions to social problems founded on altruism, conscience, and social norms as explanatory constructs, van de Kragt et al. (1983) posed the following question: "Under what circumstances will individuals who are *not* constrained by a concern for the welfare of others nevertheless act so as to provide valued public goods for the groups of which they are members?" (p. 113). Rather than attempting to answer this question by theoretical arguments, they proposed and employed a simple and attractive experimental paradigm for small group experiments in which n subjects each received a fixed monetary endowment and then chose privately whether to contribute the endowment to a monetary public good; the good is supplied if a prespecified number m or more of contributions are made. Communication among the subjects was the major independent variable in these experiments. Given the opportunity to discuss

the problem and coordinate strategies for 10 min before making their decisions privately, the subjects organized themselves by specifying the m contributors in a way that always resulted in an efficient provision of the public good. But even when predecisional discussion was prohibited and feedback not provided, the public good was provided 65% of the time; in addition, subjects overprovided more than 50% of the time, which resulted in inefficiency.

Rapoport (1985) contended that the decision whether to contribute or not depends on the size of the group ($n = 7$ in the above experiment), the size of the minimal contributing set ($m = 3$ or $m = 5$ in the above experiment), the value of the endowment and benefit (\$5 and \$10, respectively), and the probability that each group member assigns to the event that each of the other members of his or her group will contribute. van de Kragt et al. (1983) did not propose a general theory that incorporates all of these variables to account for their experimental results. And although Frohlich and Oppenheimer (1978, chap. 3) and R. Hardin (1982, chap. 4) undertook an expected value maximization analysis of the minimal contributing set (MCS) paradigm discussed above, they left the decision maker's beliefs regarding the decisions of the other group members completely unspecified. Following Straffin's (1977) probabilistic approach to the two classical power indexes formulated by Shapley and Shubik (1954) and Banzhaf (1965), Rapoport (1985) proposed alternative assumptions about the decision maker's beliefs and then examined their social implications. van de Kragt et al. (1983), Palfrey and Rosenthal (1985a), and Rapoport (1987) discussed extensions of the basic MCS paradigm, which only concern interpersonal conflict.

Intergroup and Interpersonal Competition for the Provision of Binary Public Goods

A generalization of the MCS paradigm to intergroup competition is achieved by dividing the subjects into two distinct and separate groups, A and B, and providing the public good (a monetary prize, in our case) to all the members of both groups if and only if the number of contributors in each group exceeds the number of contributors in the other group. If the number of contributors in both groups is equal, the public good is to be shared equally between the two groups. When subjects are so divided, the intergroup public good (IPG) experimental paradigm may be formulated as follows:

1. The game is played by two groups A and B with n_A and n_B members, respectively. The players are not known to each other. Within- or between-group communication before or during the game is prohibited.
2. Each player in both groups receives a fixed monetary endowment of e units ($e > 0$) and must decide independently and anonymously whether to contribute it to the benefit of his or her group.
3. If the number of contributors in Group A exceeds the number of contributors in Group B, each member of Group A receives a reward (public good) of r units. No reward is provided to the members of Group B. The same is true if the number of contributors in Group B exceeds the number in Group A. If the number of contributors in both groups is equal, each player in both groups receives a reward of s units.

Table 1
Payoff Matrix for the Intergroup Public Good Paradigm With No Communication and No Feedback

Player i's decision	Relations between numbers of contributors			
	$m_A < m_B - 1$ (1)	$m_A = m_B - 1$ (2)	$m_A = m_B$ (3)	$m_A > m_B$ (4)
Contribute (C)	0	s	r	r
Not contribute (\bar{C})	e	e	$s + e$	$r + e$

Note. m_A = number of contributors in Group A, excluding i ; m_B = number of contributors in Group B; s = number of reward units if the number of contributors in both groups is equal; r = number of public-goods reward units; e = number of fixed monetary endowment units.

4. With the exception of the final payoff, no feedback is provided about the game. In particular, a player may never learn how many players in the ingroup or outgroup have actually contributed.

Assumption 4 implies that $n_A > 2$ and $n_B > 2$. Note that if $n_A = 2$ and one of the two members of Group A contributes, a reward of zero units informs him or her that the other member of Group A did not contribute, contrary to Assumption 4. It is natural to assume that $r > s > e$. If $n_A = n_B$, it is also natural, though by no means necessary, to assume that $s = r/2$.

The major difference between the IPG and MCS paradigms is that the minimal level of contribution required for the public good to be provided is no longer determined exogenously. Rather, the outcome of the intergroup conflict, which is established by comparing the number of contributors in both groups, determines which group wins and consequently receives the public good.

Without loss of generality, assume that Player i is a member of Group A. The payoff matrix for Player i is shown in Table 1. Player i has two strategies, to contribute (C) or not contribute (\bar{C}). The assumptions $e > 0$ and $s > e$ made above imply that neither C nor \bar{C} is a dominating strategy. There are four columns in Table 1; the number of contributors in Groups A and B, excluding Player i , are denoted m_A and m_B , respectively ($0 \leq m_A \leq n_A - 1, 0 \leq m_B \leq n_B$). Column 1 refers to the case $m_A < m_B - 1$. Whether Player i contributes in this case or not, Group A will necessarily lose the competition. Column 2 refers to the case $m_A = m_B - 1$. By contributing in this case, Player i will change a losing situation into a tie. If he or she contributes, then the number of contributors in both groups will be rendered equal, and each player, including Player i , will receive a reward of s units. Column 3 refers to the case $m_A = m_B$. By contributing in this case, Player i can break the tie and lead his or her group to victory. If he or she contributes, the number of contributors in Group A will exceed the number of contributors in Group B, and therefore each member of Group A will receive r rather than s units. Finally, column 4 refers to the case $m_A > m_B$. Player i 's decision in this case—to contribute or not contribute—will obviously have no effect on the outcome of the competition.

The probabilities assigned by Player i to these four cases are

denoted P_1, P_2, P_3 , and P_4 , respectively. Clearly, $P_1 + P_2 + P_3 + P_4 = 1$. The expected value (EV) of contributing for Player i is

$$EV(C) = P_2s + P_3r + P_4r,$$

whereas the expected value of not contributing is

$$EV(\bar{C}) = P_1e + P_2e + P_3(s + e) + P_4(r + e).$$

Defining $D = EV(C) - EV(\bar{C})$ as the difference in expected value for Player i between contributing and not contributing, we obtain

$$D = P_2s + P_3(r - s) - e. \tag{1}$$

If we assume further that $s = r/2$, Equation 1 reduces to

$$D = \frac{r(P_2 + P_3)}{2} - e. \tag{2}$$

Recall that P_2 is the probability that by contributing Player i prevents his or her group from losing, whereas P_3 is the probability that by contributing Player i assures his or her group of winning. The sum $P_2 + P_3$ is the probability that Player i will make a difference in the attainment of the step-level public good for his or her group or of being critical. Another way of stating Equation 2 is to say that the expected value model predicts contribution if the probability $(P_2 + P_3)/2$ exceeds the ratio e/r of cost to benefit.

Homogeneity Model

Assumption 1 of the IPG paradigm prohibits communication before or during the game. And Assumptions 1 and 4 jointly preclude familiarity with the other players or feedback about their decisions in the single-trial task. It is most reasonable, therefore, to assume that Player i has equivalent expectations regarding the decision of each of the remaining members of his or her group, and similarly that Player i has equivalent expectations regarding the decision of each member of the outgroup. The number of parameters in our model would have been reduced if we were to assume that Player i 's expectations regarding the decisions of members of the ingroup and outgroup are the same. However, some evidence suggests that in intergroup conflicts subjects tend to believe that members of the ingroup are more likely to cooperate than are members of the outgroup (Tajfel, 1981).

Assume as before that Player i is a member of Group A and define p_j as the subjective probability of Player i that Player j will contribute toward the provision of the public good ($j \in A, j \neq i$). Define q_k as Player i 's subjective probability that Player k will contribute ($k \in B$). Then the homogeneity assumption, which was invoked in the model proposed by Rapoport (1985, 1987) for the MCS paradigm, now takes the following form: Numbers p and q are selected ($0 \leq p \leq 1, 0 \leq q \leq 1$) such that $p_j = p$ for all j and $q_k = q$ for all k .

With these two probabilities of contribution, p and q , the homogeneity assumption allows the derivation of closed form expressions for P_2 and P_3 in Equation 1 as follows:

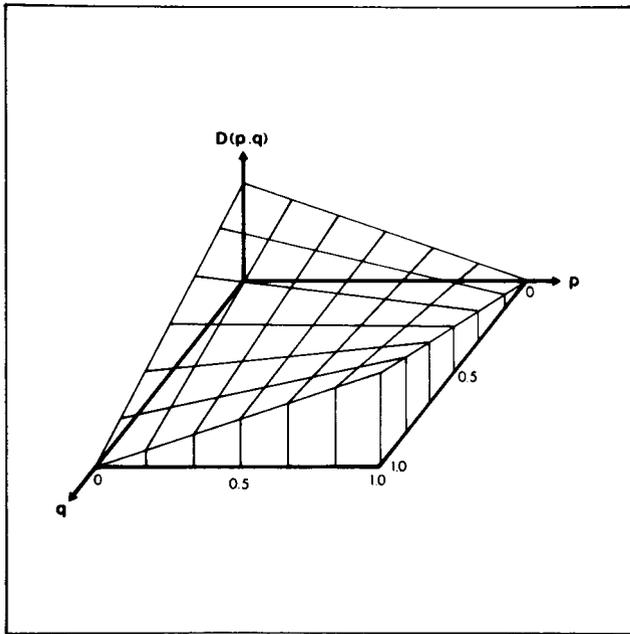


Figure 1. The difference function $D(p, q)$ for $n_A = n_B = 4$, $e = 5$, $r = 18$, $s = 9$, and $u(x) = x$.

$$P_2 = \sum_{j=0}^{n_A-1} \binom{n_A-1}{j} p^j (1-p)^{n_A-j-1} \times \binom{n_B}{j+1} q^{j+1} (1-q)^{n_B-j-1}, \quad (3)$$

and

$$P_2 = \sum_{j=0}^{n_A-1} \binom{n_A-1}{j} p^j (1-p)^{n_A-j-1} \binom{n_B}{j} q^j (1-q)^{n_B-j}. \quad (4)$$

Equations 3 and 4 both assume that $n_B \geq n_A$. The equations for the case $n_A > n_B$ are similar and are therefore omitted.

Assume that each of the two groups includes four members, that $e = 5$, $r = 18$, and $s = 9$. $D(p, q)$ is substituted for D in Equations 1 and 2 to express the dependency of the difference between the two expected values $EV(C)$ and $EV(\bar{C})$ on the two parameters of the homogeneity model, p and q . For the parameters of the present example, Equation 2 yields

$$D(p, q) = 9[p^3q^3(4 - 3q) + 6p^2(1 - p)q^2(3 - 4q + q^2) + 6p(1 - p)^2(2q + q^2)(1 - q)^2 + (1 - p)^3(1 - q)^3(1 + 3q)] - 5. \quad (5)$$

Figure 1 displays the function $D(p, q)$ for the parameters of the present example. The actual values assumed by $D(p, q)$ are not shown in order to simplify the presentation of the three-dimensional figure. Portrayed in a three-dimensional space, the surface generated by $D(p, q)$ looks like a slightly twisted saddle. The highest values of $D(p, q)$ are obtained at the two points $p = q = 0$ and $p = q = 1$, whereas the lowest values are obtained at the two points $(p = 0, q = 1)$ and $(p = 1, q = 0)$.

A special and interesting case occurs when Player i assigns the same probability to the decision of each of the other players to contribute (i.e., $p = q$). In this case, Equation 5 reduces to

$$D = 9(4[p(1 - p)^6 + p^6(1 - p)] + 18[p^3(1 - p)^4 + p^4(1 - p)^3] + 12[p^5(1 - p)^2 + p^2(1 - p)^5] + [p^7 + (1 - p)^7]) - 5.$$

The upper line in Figure 2 depicts the function $D(p, q)$ for $p = q$ and the parameters of the present example. This function (corresponding to the main diagonal in Figure 1) is bimodal, reaching its maximum of $D(p, q) = 4$ when either $p = q = 0$ or $p = q = 1$. It crosses the horizontal zero line at $p = 0.405$ and $p = 0.595$. Thus, if the ratio of cost to reward is 5:18 and each group includes four members, when assigning the same probability of contribution to each of the other members in both groups, Player i should contribute only if this probability is either above 0.595 or below 0.405.

To show the dependency of the results on the cost-to-reward ratio, suppose that the value of the public good is decreased from $r = 18$ to $r = 12$, whereas all the other parameters remain the same. Assuming again that $p = q$, the lower line in Figure 2 portrays the function $D(p, q)$ for $e = 5$, $r = 12$, $s = 6$, and $n_A = n_B = 4$. As before, the function reaches its maximum at $p = 0$ and $p = 1$ and its minimum at $p = 0.5$. However, it now intersects the zero horizontal line at $p = 0.068$ and $p = 0.932$. When r is decreased from 18 to 12 (and s is reduced from 9 to 6), a maximization of expected value policy dictates not to contribute if $0.068 \leq p \leq 0.932$ and to contribute otherwise.

Subjects participating in binary public good experiments have utilities that represent their tastes and preferences and, consequently, determine their behavior. In addition to the endowment and reward, these utilities may reflect altruism, social norms of fairness and justice, and moral or religious convictions. When utilities are substituted for values, the player's expected utility (EU) of contributing becomes

$$EU(C) = u(0)P_1 + u(s)P_2 + u(r)(P_3 + P_4) + c,$$

where $u(x)$ is the utility of x , and c is a nonnegative utility associated with the act of cooperation. Player i 's expected utility of not contributing is

$$EU(\bar{C}) = u(e)(P_1 + P_2) + u(s + e)P_3 + u(r + e)P_4.$$

Assuming with no loss of generality that $u(0) = 0$, Equation 1 then takes the form

$$D = [u(s) - u(e)]P_2 + [u(r) - u(s + e)]P_3 - [u(r + e) - u(r)]P_4 - u(e)P_1 + c. \quad (6)$$

Under the homogeneity assumption for the IPG paradigm, P_2 and P_3 are given by Equations 3 and 4, respectively, P_1 is computed from

$$P_1 = \sum_{j=0}^{n_A-1} \sum_{k=j+2}^{n_B} \binom{n_A-1}{j} p^j (1-p)^{n_A-j-1} \times \binom{n_B}{k} q^k (1-q)^{n_B-k}, \quad (7)$$

and P_4 is computed from

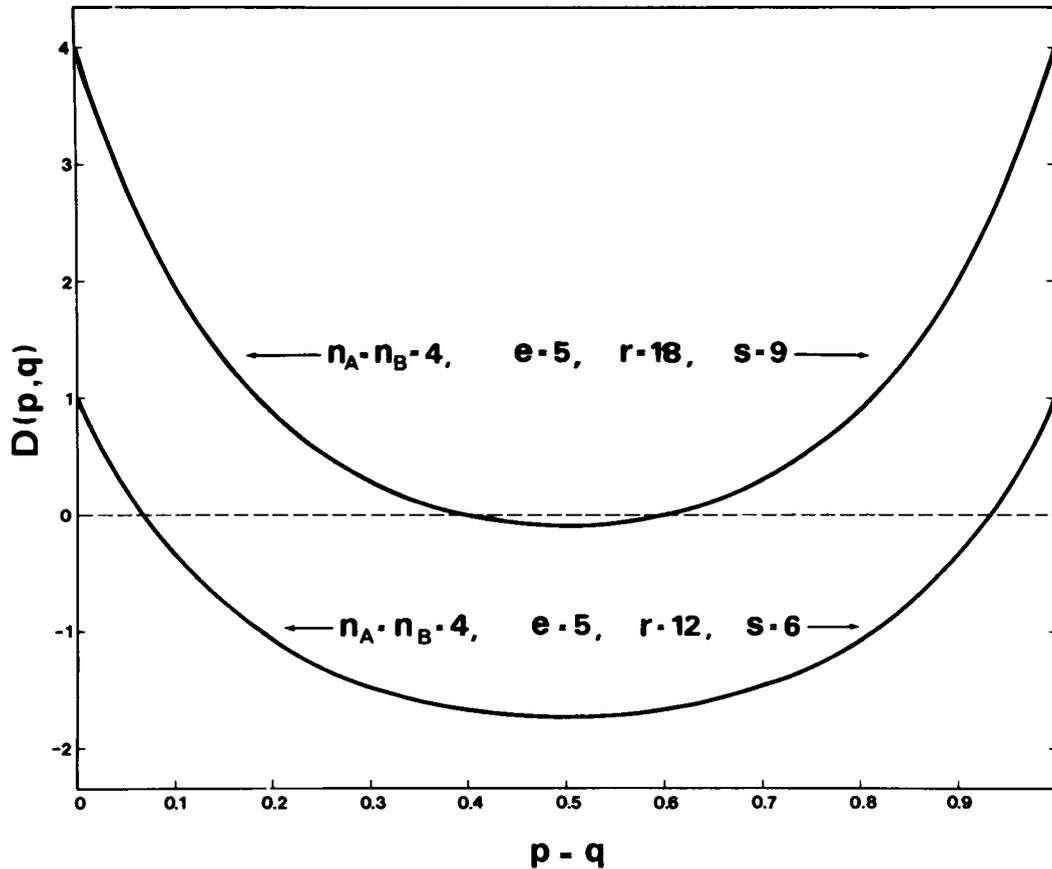


Figure 2. The difference function $D(p, q)$ plotted continuously as a function of $p = q$ ($n_A = n_B = 4, e = 5, r = 12$ or $18, s = r/2$, and $u(x) = x$).

$$P_4 = \sum_{j=1}^{n_A-1} \sum_{k=0}^{j-1} \binom{n_A-1}{j} p^j (1-p)^{n_A-j-1} \times \binom{n_B}{k} q^k (1-q)^{n_B-k} \quad (8)$$

Equations 7 and 8, as well as 3 and 4, hold for the case $n_B \geq n_A$. When $n_B < n_A$, the limits of summation in these four equations must be changed accordingly.

Heterogeneity Model

Although each player is not familiar with the other members of his or her group or with the members of the other group, there is no necessity to represent his or her belief structure by two probabilities, p and q . Players may believe that the willingness to contribute varies from one group member to another because of differences in personality, social norms, moral conventions, and utility functions. The heterogeneity model is proposed to capture such variation in beliefs by incorporating an alternative assumption about the expectations each player has about the contribution decision of each of the other players. In particular, it assumes that the expectations each player has regarding the contribution of each of the remaining members of

the ingroup are summarized by a probability density function over the closed interval $[0, 1]$. Similarly, it assumes that a player's expectations regarding the contribution of each member of the outgroup are also summarized by a (possibly different) probability density function over the same interval $[0, 1]$. Two rather than one density functions are assumed to take account of the likely difference in beliefs regarding the behavior of members of the ingroup and outgroup.

We assume that the two density functions each belong to the beta family. Although this assumption is made mainly for mathematical tractability, it is not at all restrictive (Raiffa & Schlaifer, 1961). With two free parameters, the beta density function assumes a variety of forms: uniform, bimodal, symmetric, negatively skewed, and positively skewed, thus permitting considerable flexibility in accounting for the belief structure of Player i .

Assume as before that Player i is a member of Group A and that p_j and q_k are his or her probabilities that Player j ($j \in A, j \neq i$) and Player k ($k \in B$) will contribute, respectively. Then the heterogeneity assumption, which was previously invoked in the MCS paradigm (Rapoport, 1985), takes the following form: The p_j s are selected independently from a beta density function $B(\alpha, \beta)$, and the q_k s are selected independently from a beta density function $B(\gamma, \delta)$.

Under the heterogeneity assumption, a sample of size $n_A - 1$

is drawn by a binomial sampling from a Bernoulli process whose parameter p is no longer a constant—as in the homogeneity model—but rather a random variable distributed according to the beta function with parameters $\alpha > 0$ and $\beta > 0$. Similarly, another sample of size n_B is assumed to be drawn by binomial sampling from a Bernoulli process whose parameter q is distributed according to a beta function with parameters $\gamma > 0$ and $\delta > 0$. The unconditional distributions of the number of contributions— \bar{m}_A and \bar{m}_B (which are now random variables rather than constants)—are compound mass functions known as the beta-binomial (Raiffa & Schlaifer, 1961) or Polya-Eggenberger (Johnson & Kotz, 1969). Because the two functions $B(\alpha, \beta)$ and $B(\gamma, \delta)$ are independent of each other, the probabilities P_1 , P_2 , P_3 , and P_4 in Equation 6 are obtained by multiplication of the probability mass functions and summation over the pairs of m_A and m_B , which are subsumed under columns 1, 2, 3, and 4 of Table 1, respectively (see the Appendix).

When $B(\alpha, \beta) = B(\gamma, \delta)$, heterogeneity of belief mitigates against contribution toward the provision of the public good. Following Rapoport (1985), it can be shown that when $B(\alpha, \beta) = B(\gamma, \delta)$, $D(\alpha, \beta; \gamma, \delta) \leq D(p, q)$ where $p = q = \alpha/(\alpha + \beta) = \gamma/(\gamma + \delta)$. As the parameters α , β , γ , and δ increase, the beta function $B(\alpha, \beta)$ approaches a point distribution with all the mass on its mean $p = \alpha/(\alpha + \beta)$, the beta function $B(\gamma, \delta)$ approaches a point distribution with all the mass on its mean $q = \gamma/(\gamma + \delta)$, and $D(\alpha, \beta; \gamma, \delta)$ approaches $D(p, q)$ from below.

Preplay Communication

van de Kragt et al. (1983) reported that predecisional communication in the MCS paradigm always resulted in the provision of the public good. Moreover, the public good was provided in an optimal manner by nearly all of their seven-member groups. To explain their findings, van de Kragt et al. suggested the hypothesis that subjects use the opportunity to communicate to organize themselves by specifying exactly the number of contributors required for the provision of the public good. If the designated contributing set is minimal, each member's contribution becomes critical. According to this hypothesis, members of the minimal contributing set are in a position of payers in a market of private goods. If they do not pay the designated costs, they may not enjoy the good in question. The decision to form a minimal contributing set denies the players the opportunity—indeed, the temptation—to free ride and thus ensures the provision of the public good.

The argument of criticalness is valid only if the designated contributors have firm expectations that the other members will not renege. van de Kragt et al. (1983) suggested that the structure of the game provides a reasonable basis for developing such expectations, because a designated contributor knows that his or her contribution is necessary for provision and that all the other contributors also recognize this fact. They wrote, "While contributing is not a dominant strategy for members of a minimal contributing set, it is a reasonable one if reasonableness can be expected from others. And the structure of the game says it can" (van de Kragt et al., 1983, p. 116).

In this section we relax the first assumption of the IPG paradigm by allowing unrestricted communication among the members of each group before but not during the game. As in

the original IPG paradigm, the decisions to contribute or not are assumed to be made independently and anonymously. We hypothesize that preplay communication in the IPG paradigm will have a similar effect of enhancing the level of contribution. Specifically, if the two groups are of the same size, it is hypothesized that if predecision communication leads each of the groups to act as a single unit, all the players will contribute.

Predecision communication provides the opportunity for the players in each group to coordinate their individual strategies and work together as a single team to enhance their mutual benefit. Assume that the members of each group have agreed to coordinate their strategies and work as a single team. Assume also that because of symmetry, the ingroup players believe that the outgroup players have also agreed to coordinate their individual strategies. Then the intergroup conflict for the provision of public goods is perceived by each group as a two-person non-zero-sum game between Groups A and B. When the two groups are of the same size, the payoff matrix has the form shown in Figure 3.

The assumption that $s > e$ implies that the lower right-hand cell of the payoff matrix in Figure 3 is in equilibrium. If all the members of Group A contribute, Group B may be disadvantaged by moving from the strategy n_B (right-hand column) to any other strategy. By doing so Group B will decrease its own reward and simultaneously increase Group A's reward from $n_A s$ to $n_A r$. Similarly, if all the members of Group B contribute, Group A may be disadvantaged by moving from the strategy n_A (bottom row) to any other strategy. Moreover, the lower right-hand cell is the only equilibrium in the payoff matrix. It is, therefore, reasonable to predict that it will be chosen by both groups.

If predecision communication does not result in coordination of individual strategies, the game reverts to the original IPG paradigm with no communication, where the payoff matrix for each individual player is given by Table 1.

When preplay communication results in coordination of individual strategies within each group, the game-theoretical analysis of the resulting two-person game—which is based on the notion of equilibrium—predicts that all the players in each group will contribute. Note, however, that the predicted group outcome $(n_A s, n_B s)$ is not Pareto optimal. Inspection of Figure 3 shows that it is Pareto inferior! If between-group as well as within-group preplay communication is allowed, the game-theoretical prediction of the final payoff is the Pareto optimal outcome: $(n_A(s + e), n_B(s + e))$. The game-theoretical analysis results, therefore, in a paradoxical yet readily testable prediction that when the two groups are of equal size all the players will contribute if unrestricted within-group preplay communication is allowed, but none will contribute if both within- and between-group communication is permitted.

The analysis above breaks down when the two groups are no longer of equal size ($n_A < n_B$). As in the equal-size case, preplay communication within each group is assumed to result in the coordination of individual strategies. However, the payoff matrix for the unequal-size case no longer has a pair of strategies in equilibrium. We do not know, therefore, how the intergroup conflict will be resolved. Because the strategy $m_A = 0$ guarantees to Group A a payoff of $n_A e$ units, the members of Group A may decide not to contribute (and thus retain their endow-

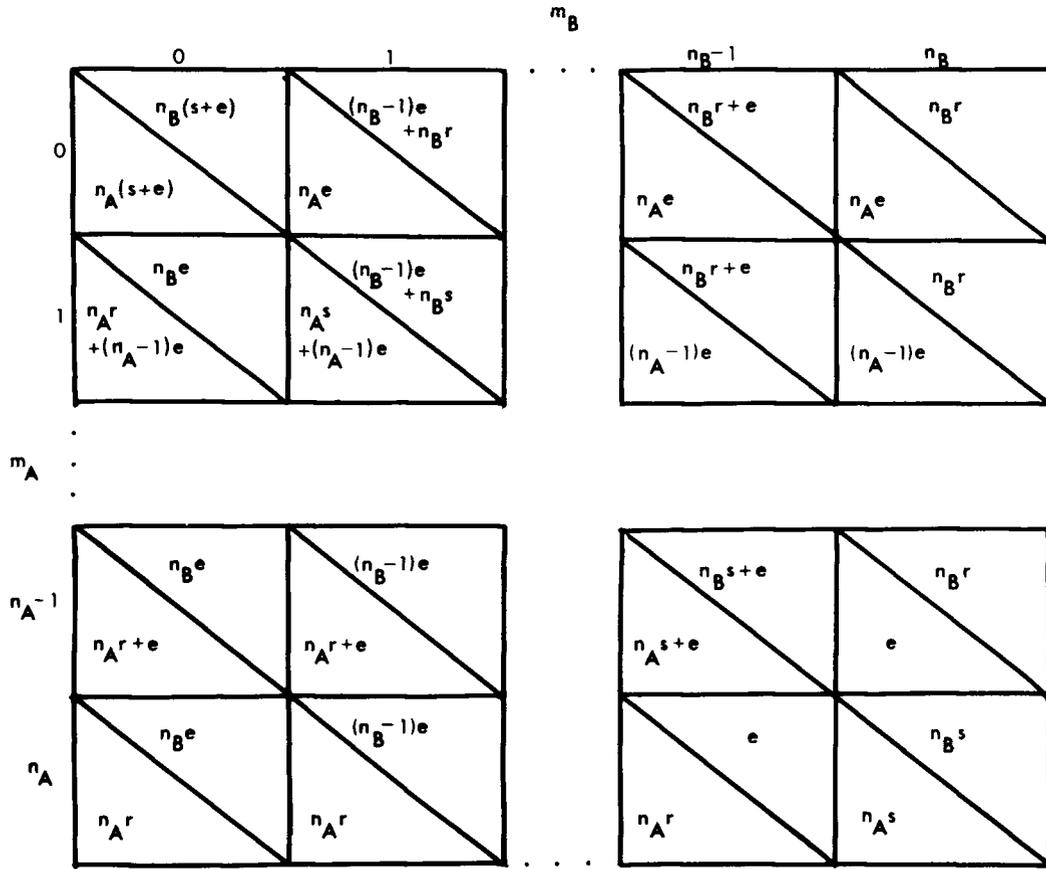


Figure 3. Payoff matrix for the intergroup public good paradigm when $n_A = n_B$ and communication results in coordination of individual strategies.

ments). The players of Group B may decide on the strategy $m_B = 1$ (only a single player contributes). However, to play it safe and protect themselves from an irrational decision on the part of Group A, they may alternatively choose the strategy $m_B = n_A + 1$ (which results in overprovision of the public good).

Discussion

Palfrey and Rosenthal (1984) noted that the free rider problem has led to two theoretical issues in the analysis of the provision of public goods. The first is the demand revelation question, which concerns the misrepresentation of individual preferences for public goods. The second issue, which can arise even when true demands are known, has to do with participation. To isolate these two issues from each other, we have followed the MCS paradigm of van de Kragt et al. (1983) and the theoretical analysis of Palfrey and Rosenthal (1983) by deliberately making all the players in both groups identical. In doing so we might have sacrificed some measure of realism for methodological convenience, because in many actual settings the contributions are not restricted to being equal, even if they are binary. To account for such settings, the IPG paradigm could be generalized by initially giving each Player i in Group G ($G = A, B$) a monetary endowment of e_i^G units ($e_i^G > 0$) and then providing the reward (public good) of r units to each member of Group

G if the sum of contributions in Group G exceeds the sum of contributions in the other group. Equality of the two sums would imply a payment of s units to each player in both groups. For example, suppose that $n_A = n_B = 4$, $e_1^A = e_1^B = 2$, $e_2^A = e_2^B = 4$, $e_3^A = e_3^B = 6$, $e_4^A = e_4^B = 8$, $r = 18$, and $s = 9$. In this example the mean endowment in each group is equal to five units, but variability in the endowments is nonzero. Complete information regarding the distribution of endowments and the magnitudes of r and s is assumed as before. Instead of restricting the investigation to voluntary contribution by financially identical players, this generalization would permit an investigation of the effects of the magnitude of within-group variation in the cost-to-benefit ratio, e_i^G/r , on voluntary contribution.

In actual settings, the contributions might be discrete or continuous and the public good itself might be discrete or continuous. Following again the works of van de Kragt et al. (1983) and Palfrey and Rosenthal (1983), we have limited the IPG paradigm to the simplest case of a binary public good with binary contributions. If sums of contributions in each group rather than numbers of contributors are to be compared between the two groups, another generalization of the IPG paradigm might still limit itself to binary public goods but allow nonbinary contributions. In this generalization, a player might be permitted to contribute any portion of his or her endowment or, to simplify the theoretical analysis, contributions might be restricted

to integral units. This generalization is subsumed under a type of game called *team game*, which has been characterized by Palfrey and Rosenthal (1983). A *participation game*—to use the terminology of Palfrey and Rosenthal again—is a special case of team games involving binary decisions by all players.

Our approach toward the analysis of participation games can be compared with the analysis of Palfrey and Rosenthal (1983), whose primary motivation was to reexamine the celebrated “paradox of not voting” (Downs, 1957; Riker & Ordeshook, 1968; Tullock, 1967). Briefly stated, the paradox claims that a rational citizen would never find it in his or her interest to vote on instrumental grounds because the probability of being the decisive voter in a large electorate is too minuscule to compensate for the cost of voting. In reexamining this issue, Palfrey and Rosenthal (1983) employed the IPG game to model a two-candidate (or proposal) election with members of A each getting a payoff of 1 if its candidate (or proposal) wins and 0 if the other candidate (or proposal) wins. The payoffs to voters in Group B are the opposite. They, too, assumed identical costs of voting to all the players, a payoff of 1/2 if the number of voters in both groups is equal, and simultaneous choices. Voters were assumed to be narrowly self-interested, attaching no sense of citizen duty or private consumption value to the act of voting. It is easily seen that this model of a two-candidate election is identical to the IPG game with $r = 1$, $s = 1/2$, and $0 < e < r$. The theoretical analysis of Palfrey and Rosenthal consisted of finding all the equilibria in the game. Showing that multiple equilibria proliferate, they could make no strong predictions about the size of the voter turnout (number of contributors). However, for large electorates, Palfrey and Rosenthal discovered only two types of equilibria that involve either zero voters or a number of voters very close to twice the size of the smallest group ($2 \min [n_A, n_B]$). (Note that for $n_A = n_B$, this is 100%.)

The approach taken in this article to model the players' decisions in the IPG game differs from the analysis of Palfrey and Rosenthal (1983) in several important respects. First, in both the homogeneity and heterogeneity models consistency in expectations should lead the players other than i to assume that Player i also has a probability of contribution. However, both models assume that this probability is either 0 or 1. Therefore, both models assume that players have incorrect expectations. A game-theoretic approach such as the one taken by Palfrey and Rosenthal would attempt to complete the models by viewing the IPG paradigm as an incomplete information game and then characterize the Bayesian equilibria (Ledyard, 1986; Palfrey & Rosenthal, 1985b) of such games. We do not pursue this approach here because the game-theoretic approach may prove inadequate when the IPG game is played only once and without communication. An individual-choice behavior approach, like the one proposed in this study, may well assume that the player's expectations are incorrect. Second, both the homogeneity and heterogeneity models have been primarily proposed to account for experimental results from IPG games with a small number of players, whereas the equilibrium analysis of Palfrey and Rosenthal, to the extent that it is experimentally testable, is primarily applicable to large groups.

Experimental testing of the homogeneity and heterogeneity models requires for each player the assessment of both the subjective probability distribution over p_i and the utilities $u(r)$, $u(e)$,

$u(r + e)$, $u(s + e)$, and c . Direct and indirect procedures for assessing probabilities and measuring utilities are available, and their properties are known. The assessment of the subjective probabilities of contribution will either determine which of the homogeneity and heterogeneity models is more adequate or lead to the rejection of both. If one of the two models is rendered adequate, the individual utilities would be employed to predict for each player the decision to contribute or not.

Additional tests of the models are based on experimental manipulation of the task parameters and do not require the assessment of probabilities and utilities. Thus, both models predict an increase in the number of contributors when the ratio r/e is increased but the group sizes n_A and n_B are kept constant. Both models also make the sensible prediction that if $n_A = n_B = n$, the number of contributors will get smaller as n is increased. Finally, the effects of preplay communication on the number of contributors could be tested by allowing unrestricted within-group communication or permitting both within- and between-group communication before requiring the players to make their binary decisions independently and anonymously.

References

- Banzhaf, J. F., III. (1965). Weighted voting doesn't work: A mathematical analysis. *Rutgers Law Review*, 19, 317–343.
- Barry, B., & Hardin, R. (Eds.). (1982). *Rational man and irrational society?* Beverly Hills, CA: Sage.
- Billig, M. (1976). *Social psychology and intergroup relations*. London: Academic Press.
- Brewer, M. B. (1979). In-group bias in the minimal intergroup situation: A cognitive-motivational analysis. *Psychological Bulletin*, 86, 307–324.
- Brewer, M. B., & Kramer, R. M. (1986). Choice behavior in social dilemmas: Effects of social identity, group size, and decision framing. *Journal of Personality and Social Psychology*, 50, 543–549.
- Campbell, D. T. (1975). On the conflicts between biological and social evolution and between psychology and moral tradition. *American Psychologist*, 30, 1103–1126.
- Downs, A. (1957). *An economic theory of democracy*. New York: Harper & Row.
- Frohlich, N., & Oppenheimer, J. A. (1978). *Modern political economy*. Englewood Cliffs, NJ: Prentice-Hall.
- Gerard, H. B., & Miller, N. (1967). Group dynamics. *Annual Review of Psychology*, 18, 287–332.
- Hardin, G. (1968). The tragedy of the commons. *Science*, 162, 1243–1248.
- Hardin, G. (1977). *The limits of altruism*. Bloomington: Indiana University Press.
- Hardin, R. (1982). *Collective action*. Baltimore, MD: Johns Hopkins University Press.
- Johnson, N. L., & Kotz, S. (1969). *Distributions in statistics: Discrete distributions*. Boston: Houghton Mifflin.
- Kramer, R. M., & Brewer, M. B. (1984). Effects of group identity on resource use in a simulated common dilemma. *Journal of Personality and Social Psychology*, 46, 1044–1057.
- Ledyard, J. O. (1986). The scope of the hypothesis of Bayesian equilibrium. *Journal of Economic Theory*, 39, 59–82.
- Palfrey, T. R., & Rosenthal, H. (1983). A strategic calculus of voting. *Public Choice*, 36, 7–53.
- Palfrey, T. R., & Rosenthal, H. (1984). Participation and the provision of discrete public goods: A strategic analysis. *Journal of Public Economics*, 24, 171–193.

Palfrey, T. R., & Rosenthal, H. (1985a). *Private incentives in social dilemmas*. Unpublished manuscript.
 Palfrey, T. R., & Rosenthal, H. (1985b). Voter participation and strategic uncertainty. *American Political Science Review*, 79, 62-78.
 Rabbie, J. M. (1982). The effects of intergroup competition and cooperation on intragroup and intergroup relationships. In V. J. Derlega & J. Grzelak (Eds.), *Cooperation and helping behavior: Theories and research* (pp. 123-149). New York: Academic Press.
 Raiffa, H., & Schlaifer, R. (1961). *Applied statistical decision theory*. Cambridge: MIT Press.
 Rapoport, A. (1985). Provision of public goods and the MCS paradigm. *American Political Science Review*, 79, 148-155.
 Rapoport, A. (1987). Research paradigms and expected utility models for the provision of public goods. *Psychological Review*, 94, 74-83.
 Riker, W., & Ordeshook, P. (1968). A theory of the calculus of voting. *American Political Science Review*, 62, 25-42.
 Shapley, L. S., & Shubik, M. (1954). A method for evaluating the distri-

bution of power in a committee system. *American Political Science Review*, 48, 787-792.
 Straffin, P. D., Jr. (1977). Homogeneity, independence, and power indices. *Public Choice*, 30, 107-118.
 Tajfel, H. (1981). *Human groups and social categories*. Cambridge, England: Cambridge University Press.
 Tajfel, H. (1982). Social psychology of intergroup relations. *Annual Review of Psychology*, 33, 1-39.
 Tullock, G. (1967). *Towards a mathematics of politics*. Ann Arbor: University of Michigan Press.
 Turner, J. C. (1975). Social comparison and social identity: Some prospects for intergroup behavior. *European Journal of Social Psychology*, 5, 5-34.
 van de Kragt, A. J. C., Orbell, J. M., & Dawes, R. M. (1983). The minimal contributing set as a solution to public goods problems. *American Political Science Review*, 77, 112-122.

Appendix

State Probabilities Under the Heterogeneity Assumption

The difference in expected utilities between the decisions to contribute or not is given by

$$D = [u(s) - u(e)]P_2 + [u(r) - u(s + e)]P_3 - u(e)P_1 - [u(r + e) - u(r)]P_4 + c,$$

where $u(x)$ is the utility of x and c is the utility associated with the act of contribution.

Assuming that $n_A \leq n_B$ and that the parameters $\alpha, \beta, \gamma,$ and δ are positive integers, the heterogeneity model yields the following expressions:

$$P_1 = \sum_{j=0}^{n_B-1} \sum_{k=j+2}^{n_B} \frac{(j + \alpha - 1)!(\beta + n_A - j - 2)!(n_A - 1)!(\alpha + \beta - 1)!}{j!(\alpha - 1)!(n_A - j - 1)!(\beta - 1)!(\alpha + \beta + n_A - 2)!} \times \frac{(k + \gamma - 1)!(\delta + n_B - k - 1)!n_B!(\gamma + \delta - 1)!}{k!(\gamma - 1)!(n_B - k)!(\delta - 1)!(\gamma + \delta + n_B - 1)!};$$

$$P_2 = \sum_{j=0}^{n_A-1} \frac{(j + \alpha - 1)!(\beta + n_A - j - 2)!(n_A - 1)!(\alpha + \beta - 1)!}{j!(\alpha - 1)!(n_A - j - 1)!(\beta - 1)!(\alpha + \beta + n_A - 2)!} \times \frac{(j + \gamma)!(\delta + n_B - j - 2)!n_B!(\gamma + \delta - 1)!}{(j + 1)!(\gamma - 1)!(n_B - j - 1)!(\delta - 1)!(\gamma + \delta + n_B - 1)!};$$

$$P_3 = \sum_{j=0}^{n_A-1} \frac{(j + \alpha - 1)!(\beta + n_A - j - 2)!(n_A - 1)!(\alpha + \beta - 1)!}{j!(\alpha - 1)!(n_A - j - 1)!(\beta - 1)!(\alpha + \beta + n_A - 2)!} \times \frac{(j + \gamma - 1)!(\delta + n_B - j - 1)!n_B!(\gamma + \delta - 1)!}{j!(\gamma - 1)!(n_B - j)!(\delta - 1)!(\gamma + \delta + n_B - 1)!};$$

$$P_4 = 1 - P_1 - P_2 - P_3.$$

If the parameters $\alpha, \beta, \gamma,$ and δ are assumed to be positive real numbers rather than positive integers, the factorials in the above expressions ought to be replaced by the gamma function, which is discussed in books on calculus ($\Gamma(x) = (x - 1)!$, if x is a positive integer).

If $n_A > n_B$, the summations in the above expressions ought to be replaced by $\sum_{j=0}^{n_B-2} \sum_{k=j+2}^{n_B} \sum_{j=0}^{n_B-1} \sum_{j=0}^{n_B}$, respectively.

Received April 1, 1986
 Revision received October 14, 1986
 Accepted October 17, 1986 ■